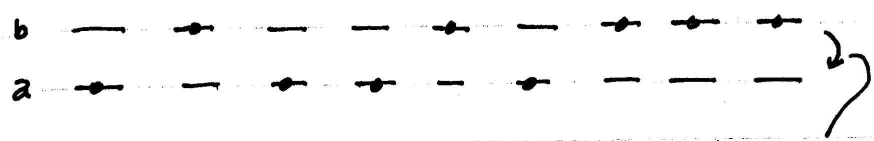
Spontaneous Emission

N_b atoms where e^- is in the "higher energy" state



N_b

N_a

$b \to a \quad \frac{\#}{\text{time}} = N_b A$ } Spontaneous Emission

<u>Stimulated Emission</u> $\quad R_{b \to a} = \frac{\pi}{3\varepsilon_0 \hbar^2} |\wp|^2 \rho(\omega_0)$ } incoherent unpolarized all directions

$b \to a \quad \frac{\#}{\text{time}} = N_b B_{ba} \rho(\omega_0)$ } Stimulated Emission

$$\boxed{B_{ba} = \frac{\pi}{3\varepsilon_0 \hbar^2} |\wp|^2}$$

Absorption Rate $\quad a \to b \quad \frac{\#}{\text{time}} = N_a B_{ab} \rho(\omega_0)$

In an ambient field of e.m. radiation

$$\frac{dN_b}{dt} = -N_b A - N_b B_{ba} \rho(\omega_0) + N_a B_{ab} \rho(\omega_0)$$

If the atoms are in thermal equilibrium with the ambient field, then

$$0 = -N_b A - N_b B_{ba} \rho(\omega_0) + N_a B_{ab} \rho(\omega_0)$$

$$\rho(\omega_0) = \frac{A}{\frac{N_a}{N_b} B_{ab} - B_{ba}}$$

Compare this to Planck Black Body

Spontaneous Emission

First of all, from statistical mechanics, we have the Boltzmann factor

$$\frac{N_a}{N_b} = \frac{e^{-E_a/kT}}{e^{-E_b/kT}} = e^{(E_b-E_a)/kT} = e^{\hbar\omega_0/kT}$$

So, $\rho(\omega_0)$ becomes: $\rho(\omega_0) = \frac{A}{e^{\hbar\omega_0/kT} B_{ab} - B_{ba}}$

Compare this to Black Body Radiation: $\rho(\omega) = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar\omega/kT} - 1}$

Comparing these two equation we see that

spont. emission rate

$$\boxed{B_{ab} = B_{ba}} \quad \text{and} \quad A = \frac{\omega_0^3 \hbar}{\pi^2 c^3} B_{ba}$$

$$A = \frac{\hbar\omega_0^3}{\pi^2 c^3}\left(\frac{\pi |\wp|^2}{3\epsilon_0 \hbar^2}\right) \qquad \boxed{A = \frac{\omega^3}{3\pi\epsilon_0 \hbar c^3} |\wp|^2}$$

Spontaneous Emission Rate.

Homework Problems: 9.8 9.10 9.11

<u>9.11</u> You need to calculate $|\wp|^2$ where $\wp = \langle \text{final} | e\vec{r} | \text{initial} \rangle$
and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ $\wp = e \langle \text{final} | x\hat{i} + y\hat{j} + z\hat{k} | \text{initial} \rangle$

z matrix element $\wp \to 0$ except for $\langle 100 | z | 210 \rangle = \frac{2^8}{3^5\sqrt{2}} a$

$|100\rangle$ $|200\rangle$ and $|210\rangle$ are even in (x,y)

Spontaneous Emission

Problem 9.11 continued.

So the only other non-zero matrix elements are:

$$\langle 100 | x | 2\,1 \pm 1 \rangle \quad \text{and} \quad \langle 100 | y | 2\,1 \pm 1 \rangle$$

$$\omega = \frac{E_2 - E_1}{\hbar} = -\frac{3E_1}{4\hbar}$$

<u>Calculate A</u>

The lifetime is $\tau = \dfrac{1}{A}$